FEM-BEM coupling for wave propagation problems in unbounded domains

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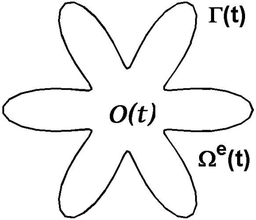
Introduction and motivation

We want to study the effect that a moving obstacle has on the behavior of waves which impinge upon it.
Such a situation occurs in many practical contexts:

- rotating blades of an helicopter, the presence of which has to be radar detected;
- signals reflected by wind turbines;
- computation of flows about rotating components (propellers).

Problem setting

Let $\Omega^e(t) = \mathbb{R}^2 \setminus \overline{\mathcal{O}}(t)$ be the complement of a bounded rigid obstacle $\mathcal{O}(t) \subset \mathbb{R}^2$, whose location depends on t, and having a smooth boundary $\Gamma(t)$.



We denote by Ω_0^e the initial configuration of the geometry at time $t = 0$. We consider the following **exterior** model problem:

$$
\begin{cases}
u_{tt}(\mathbf{x}, t) - \Delta u(\mathbf{x}, t) & = f(\mathbf{x}, t) \quad \text{in } \Omega^e(t) \times (0, T) \\
u(\mathbf{x}, t) & = 0 \quad \text{on } \Gamma(t) \times (0, T) \\
u(\mathbf{x}, 0) & = u_0(\mathbf{x}) \quad \text{in } \Omega_0^e \\
u_t(\mathbf{x}, 0) & = v_0(\mathbf{x}) \quad \text{in } \Omega_0^e.
\end{cases}
$$

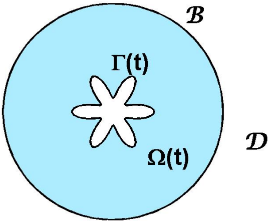
To solve it by a finite element method

1) we truncate the infinite external domain by an **artificial boundary** $\mathcal{B}$;
the artificial boundary divides $\Omega^e(t)$ into two subdomains:

- a finite computational domain $\Omega(t)$ (bounded internally by $\Gamma(t)$ and externally by $\mathcal{B}$) We impose on $\mathcal{B}$ the **exact non reflecting boundary condition** given by the Boundary Integral Equation:

$$\frac{1}{2}u(\mathbf{x},t) = \boldsymbol{\mathcal{V}}\lambda_{\mathcal{B}}(\mathbf{x},t) - \boldsymbol{\mathcal{K}}u(\mathbf{x},t) \quad \mathbf{x} \in \mathcal{B},$$

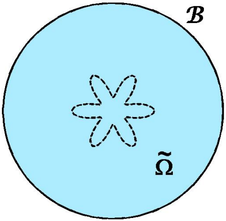
- an infinite residual domain $\mathcal{D}$ (that does not depend on t)



The fictitious domain-Lagrange multiplier formulation

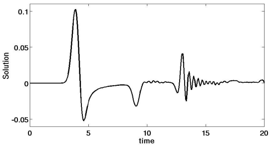
The **fictitious domain method** consists in:

- extending "artificially" the solution of the exterior problem inside the obstacle;
- solving the new problem in the whole extended domain $\widetilde{\Omega} := \Omega(t) \cup \mathcal{O}(t)$;
- enforcing the Dirichlet boundary conditions by Lagrange multipliers.

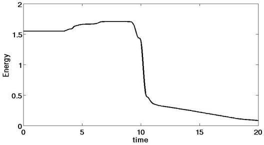


Ex: Scattering of a wave by two rotating blades

Solution at $P \approx (0, 4)$



Energy dissipation



S. Falletta, G. Monegato. *A fictitious domain approach for wave propagation problems in unbounded domains*.
S. Falletta, *BEM coupling with the FEM-fictitious domain approach for the solution of the exterior Poisson problem and of the wave scattering by rotating rigid bodies*.